American Journal of Educational Research, 2015, Vol. 3, No. 12B, 1-5
Available online at http://pubs.sciepub.com/education/3/12B/1

DOI:10.12691/education-3-12B-1

Non-verbal Shells of the Instructional Mathematical Content

Tarasenkova N. A.*

Bohdan Khmelnytsky National University of Cherkasy, Cherkasy, Ukraine
*Corresponding author: ntaras7@ukr.net

Abstract The article is an exposure to semiotic-symbolic peculiarities of non-verbal means which are the images of geometric shapes in school math, analytical configurations of some facts which are under study in secondary schools. The article details on their array and use in the learning process.

Keywords: *school math, semiotic-symbolic peculiarities, non-verbal means, geometric figures, formulas*

Cite This Article: Tarasenkova N. A., "Non-verbal Shells of the Instructional Mathematical Content." *American Journal of Educational Research*, vol. 3, no. 12B (2015): 1-5. doi: 10.12691/education-3-12B-1.

1. Introduction

All means of the objectification of the perfect, learning content included can be attributed to the concepts of "sign" and "symbol". These concepts are the subject of study of various sciences – epistemology, logic, linguistics, semiotics, and psychology), that are trying to find the solutions to a number of issues related to the nature of cognition and the ways of life of those who try to cognize the world. Every science offers its own approach to the disclosure of the cognitive concepts and their functional application. At present there is no unique, agreed-on, and psychology-backed interpretation of the term "*sign*", which is important for the theory and methods of teaching. The idea of "symbol" is dissimilar either [1,4].

For us the results of psychological and semiotic analysis of signs and symbols as well as their use in teaching and learning are of primary relevance and importance [11]. In terms of functional approach to understanding the essence of the concepts of "sign" and "symbol", which has roots in dialectical logic, signs indicate the contents of a knowledge, whereas symbols reveal the character of this knowledge (portray it, express the relation of the subject to it, etc.). According to E. Ilyenkov [2], functional character of the existence of a symbol lies in the fact that it is a means and an instrument for identifying the nature of other things that can be sensed, that is – the nature of the common. N. Salmina [6] emphasizes it that functional distinctions between signs and symbols are not that essential, so she titles their array to be a set of semantic-symbolic means (SSM). We shall rely on these regulations.

Modern classification of semantic-symbolic means (SSM) is based on four main criteria: 1) attributive qualities of the SSM form attribute (what exactly is used as a form of the sign – (substrate-substantive (essential) features); human actions; spatial characteristics); 2) the relations between the form of the SSM and the contents they designate in terms of arbitrariness / motivation (by agreement or as a result of certain properties of the content); 3) function of the form as regards contents (whether the form indicates, reveals, depicts or expresses the attitude to the contents); 4) what exactly is signified: whether a real object (phenomenon) is reproduced or the essence of something is revealed, designated or signified. In general, among the semantic-symbolic means they differentiate two classes - verbal and nonverbal SSM (M. Hamezo, N. Salmina, etc.). Each class of semantic-symbolic means is further divided into types.

2. Classification of Non-verbal SSM: Semiotic Dimension

Psycho-semiotic classification of non-linguistic (non-verbal) semantic-symbolic means used in educational practices is quite extensive. The following three types are differentiated [6] among them: 1) spatial SSM which help reveal significant relationships, structure of things and events in scientific and academic cognition or real phenomena are reproduced; 2) substrate-substantive (essential) SSM used to signify specific properties of objects or reproduce their physical or functional properties through large-scale deformation; 3) human actions that reflect some real phenomena or events and express one's attitude to them.

Each type of non-verbal SSM is divided into two subtypes - iconic SSM and arbitrary SSM. They differ by the parameters of arbitrariness/motivation in choosing a substitute. Iconic non-verbal SSM mostly reproduce visual characteristics of the signified and through these characteristics fix semantic analogies. There are no visual connections between the signified objects, phenomena or events and the substitutes when the latter are to be treated as arbitrary non-verbal SSM. In this case the choice of substitutes is made by agreement based on it that there should be some certain semantic analogy.

Spatial semantic-symbolic means (both iconic and arbitrary) are further subdivided into subspecies, namely -

two-dimensional and three-dimensional SSM, depending on the respective spatial characteristics of the substitute.

Thus, in teaching and learning mathematics the following SSM represent the class of the non-verbal semantic-symbolic means [7,8]:

In teaching mathematics the class of non-linguistic sign and symbol means is represented by the following SSM:

• images of geometric shapes, which may be used as visually plausible and visually distorted images, illustrations of different types (two-dimensional spatial iconic SSM);

• layouts and designs that, in simulated situations, will pass for modeled objects, phenomena and events of the real world (three-dimensional spatial iconic SSM);

• tables, charts, diagrams, and schematic graphics as a means of mapping structured entities (arbitrary two-dimensional spatial SSM);

• real items used in substitution functions (arbitrary three-dimensional spatial SSM);

• content and graphic interpretation of mathematical concepts, facts and ways of activity rate (intrinsic iconic SSM);

• natural language texts and mathematical sentences specifically placed on the plane (analytical configurations), content, visual and content and visual accents (intrinsic arbitrary SSM);

• plastic display of the essence of mathematical concepts, facts, or ways of life, such as the rules of conventional division of fractions, monotonic functions, perpendicular lines, dramatization, the game (human actions as iconic SSM);

• non-verbal communication, rituals (human actions as arbitrary SSM).

3. Classification of Nonverbal SSM: The Didactic Aspect

Considering the non-linguistic semantic-symbolic means in terms of their didactic purpose in teaching mathematics, we obtain a different classification [7].

Thus, it is advisable to include in the first group those SSM that will perform the function of the objects of assimilation. These are the images of geometric shapes; content and graphic interpretation of concepts, facts, and ways of life studied in the mathematics course; tables, charts, diagrams, and schematic graphics.

The second group should include those SSM that mostly perform a supporting function in teaching and learning mathematics. These are the rest of the named above non-verbal SSM including instructional illustrations.

The second group requires some reorganization. Thus, it is advisable to view three-dimensional spatial SSM (real objects, models, and designs) in their unity for in the educational process they are used in their functional unity.

In our opinion, such SSM as human action should not be separated from each other. A combined group of such means is called plasty. In their turn, the plastic means fixing mathematical content should be considered together with such iconic representatives of the two-dimensional spatial SSM as illustrations. In the illustrations for the mathematics manuals they often demonstrate the content and sequence of certain human actions.

Let us detail on some of the peculiarities of non-verbal SSM.

4. Graphic and Content-graphic Interpretation as Non-verbal SSM

The information about the spatial form of planimetric and stereometric objects and the corresponding geometric concepts is reflected not only the definition and terms, but in some graphics shells – geometric figures that are performed in a certain plane. By their semiotic properties, geometric images are non-verbal semantic-symbolic means related to two-dimensional spatial iconic SSM [7,8,10].

As such SSM through their shape and features express definite semantic features of geometric shapes (e.g.: topological, projective properties), the imaging of geometric shapes can be otherwise called *graphical interpretations* of these concepts. In our opinion, the term "geometric configuration" is also possible, though with certain reservations.

The thing is that in Geometry the notion of configuration bears a special meaning that is different from the general understanding of this concept. The term "configuration" comes from the Latin *configuratio*, which means an outer contour, a relative disposition of some items. It is in this sense that it is used in various fields of human knowledge. But in Geometry configuration, take on the plane, is a system of $\boldsymbol{h}$ points and $\boldsymbol{m}$ straight lines placed so that each point passes through the same number ($\boldsymbol{k}$) of straight lines and the same number ($\boldsymbol{e}$) of points [3,5].

In the school course of mathematics, a scientific concept of geometric configuration is not studied. However, each image of a geometric figure refers to an outer contour pattern and a relative position of its elements – points, segments, lines, circles, arcs, etc. Consequently, the use of the term "*configuration*" in its conventional meaning is quite possible here. As in teaching and learning mathematics this term can also be applied in respect of the relative position of expressions, equations, inequalities, and other elements of recording in natural or mathematical language, it is the notion of "geometric configuration" but not "configuration" as it is, is a more appropriate name to signify the notion of "image of a geometric figure".

In teaching mathematics the images of geometric figures (their graphic interpretation; geometric configuration) act as a subject of study (in the Geometry study in particular) and in the supporting functions as a visual support in cognitive and transformative activities of students. In both cases, at least three factors are essential. First, it is important how geometric configurations are introduced in instruction – whether they are given ready-made or are created by the students themselves (following certain rules or on their own). Second, it is of importance what type images are performed – accurate or close to accurate, methods for their construction, and the necessary sets of tools. And thirdly, it is also rather important, which image is processed ("read") by the student, namely – whether this image is visually authentic or visually distorted, since in the latter case, conflicts between the logical and the visual are inevitable. Each of these characteristics and their complex must be considered in educational planning.

Geometric figures often reflect, in certain agreed notations (symbols), the information about the properties of shapes that form a depicted configuration. For example, special symbols are used for denoting a right angle; the same number of strokes is used as a notation for equal segments, and the same number of brackets stands for equal angles. This figure holds not only the image, but also logical and semantic information. Therefore, it is advisable to call it *content-graphic interpretation.* Figure 1 - Figure 2 offer the examples of graphic interpretations, and their respective content and graphic interpretations.

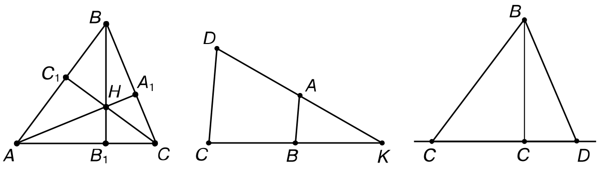

Figure 1. Graphic interpretations

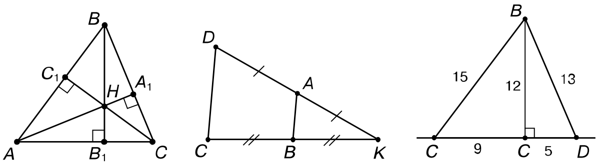

Figure 2. Content-graphic interpretations

Content-graphic interpretation of any mathematical concept or fact is a materialized convolute semantic structure [10]. If the students possess some mathematical experience, the fact of identifying the fact in the frames of the content-graphic interpretation is carried out at the highest coding level – the semantic one. The fact of the encoded content being materialized eliminates the necessity to constantly keep this content in memory in the process of analysis (say, the conditions of the problem and its solution) – it can be referred to at any time. The more mathematical facts are materialized on the content and graphic interpretation, the easier and faster the analysis of the problem and finding the solution can be. In this case the relevant experience of the students is of importance.

However, our observations show that it is not always that in school practice teachers and students seek the creation of content and graphic interpretations of geometric concepts and facts. Graphic interpretations are used most often. We believe here lies one of the reasons for poor performance in the analytic-synthetic activity of students in acquiring and applying geometric knowledge, skills and abilities, in problem solving in particular. Let us explain this.

Support in analyzing the problem only in terms of graphical interpretation requires from the students a considerable tension of the random access memory, which should hold information not only about the data of the problem, but also about the consequences of them, should form various combinations of mathematical facts, choose the most suitable ones, etc. Using the textual conditions of the problem or its short track as a means of maintaining the external content, to our knowledge, does not simplify the problem, because in that case repeated transitions from visual perception to the graphic text and vice versa are required. But graphics and text messages are processed according to different patterns and at different speeds. As a result, if, nonetheless, intermediate convolute semantic structures necessary for solving problems are formed in the students' memory, the time of their retention in the random access memory is not enough for many students to solve the problem. In these conditions these structures are destroyed much quicker than their requisite meaning is singled out and fixed. It is clear that the reliance on a semantic-graphical interpretation (even the one that is not devoid of certain flaws) simplifies the problem.

In processing semantic and graphic interpretations handling visualized logical data and manipulating them often occurs without deploying their content in a verbal form. In other words, it is visual thinking that steps on the foreground here. Thus, a purposeful development of students' appropriate mental techniques is to be considered one of the most important tasks of teaching mathematics in school.

Our study [7] shows that a didactically balanced approach to the formation and development of visual thinking gives students additional opportunities for gradual improvement of their verbal-logical thinking. In this case semantic and graphical interpretation of mathematical concepts and facts is one of the most important means of teaching.

5. Analytical Configurations as Non-verbal SSM

To indicate a number of concepts and facts of the mathematics school course alphanumeric symbols are used, and for a convolute reification and coding they use mathematical sentences, which are content and analytical interpretations of concepts and facts. It is not only the content of such sentences which is of importance for the high school students, but the topography of the corresponding entries – outlines and relative position of their elements. Due to this factor the recordings of expressions, equations, inequalities, and other text elements created by means of natural or mathematical language, acquire a specific shape, which should be called *analytical configuration*.

5.1. Topography of Analytical Configuration and Its Impact on Understanding and Assimilation of the Content

Due to their certain topography analytical configuration can be referred to the non-verbal SSM, namely – to the class of arbitrary essential SSM. In teaching mathematics there should be taken into account the fact that these arbitrary essential SSM are the objects of acquisition learning and they directly or indirectly affect the efficiency of the process.

For example, to indicate a particular negative number a positive number, contrary to this, is used, the sign "minus" being put in front of it:

$$-1;\ -\frac{23}{45};\ -6,789.$$

A record:

$$\text{«}-a \text{ if } a > 0 \text{ or } a \text{ if } a < 0\text{»} \qquad (1)$$

is a content-analytical interpretation of the concept of a negative number.

Experience shows that the concept of negative numbers stays unformed if students do not realize the unity of the two components of this mathematical sentence (*–a* if *a* > 0; *a* if *a* <0). If in the students' experience there does not exist the idea of the content and analytical interpretation of negative numbers as a kind of their code, it may cause many difficulties and mistakes in mastering and using the other algebra course concepts – number module, the square root of the number, the logarithm of the number, etc.

However, since the concept of a negative number is entered in the 6th grade (the 6th year of study in high school) through examples without any strict definition, the majority of students identify a negative number with any number (or letter, that indicates it in general) with a sign "–" before it. Thus, for the schoolchildren, "–" becomes the only indicator for the negative number and it suffices.

So, some visual topographic image (VTI) is spontaneously formed in students' minds and can be represented by the following configuration:

« – þ ». (2)

In this configuration there are two important visually defined and separated components, whose topography (outline and relative dislocation) in this visual series is quite defined and rigid.

Generally, we view VTI of certain concepts as a student's subjective formation reflecting some analytical configuration in which the students themselves accentuate certain details. In the process of visual recognition the analytical configuration is an object of visual perception. But it is the VTI that has been formed in/through the experience of students that plays the role of the original stimulus. The student allegedly instantly compares what they see with what is stored in their memory.

Visual topographical image may or may not coincide with a particular interpretation of the concept. Consistencies (matching) arise when graphical or content and graphic interpretation of concepts is employed in analytical configuration. Divergences occur when semantic-analytical interpretations are used. In the cases of consistency VTI is adequate to the concept. In student's experience the VTI serves as a shell for the code structure of the concept, hence the recognition meets no flaws. A partial visual topographic image cannot be adequate to the concept. It often plays a provoking role in visual analysis. Students mostly cannot recognize the concept on their own.

For example, an analytical configuration (2) corresponds to a partial VTI of a negative number. However, the appearance of this VTI in the experience of the students is supported by the visual series which is formed under the traditional design of recording (1). In this recording only the sign "–" that goes first is visually separated from the others. The rest of the recording visually merges. Thus, in the topography of the recording its essential components should not be just isolated, they should visually be perceived as separate ones.

In our view, any linguistic material, and particularly the one meant for learning must undergo "configuration". It is also of importance that students should directly participate in the creation of an analytical configuration.

5.2. Matrix Notations of Entries

One of the ways of creating analytical configurations is the matrix registration of the records created by means of natural or mathematical language [14]. Matrix notation of entries is preferable as content analysis is always preceded by a visual one, and visual comparison is easier to do vertically than horizontally. The fact is that while solving equations they preferably use the column arrangement. This way the visual (and, as a consequence – the content one) analysis of transformations of certain expressions gets easier. For example, for the awareness and the assimilation of the operations of subtraction and addition in parts, Matrix notations of entries will come in handy.

$$\begin{array}{rcccl} & 6 & + & 8 & = \\ = & (6+4) & + & (8-4) & = \\ = & 10 & + & 4 & =14 \end{array} \qquad (3)$$

As it has been already mentioned, within representation and operating analytical, and content and analytical interpretations of concepts, facts and ways of working, the students get an idea of a certain visual topographic image which, in many cases, appears to be inadequate to the content of the corresponding object of assimilation. This inadequacy is most vividly manifested when this content is wrapped in a verbal semantic-symbolic shell.

Within the research we found out in this case it is matrix notations of entries that helps. For example, when the concept of a negative number is introduced, pupils face much less listed above difficulties when the corresponding analytical interpretation is arranged in the following way [13]:

«– *a* if *a* > 0

or

a if *a* < 0». (4)

Let's turn our attention to those situations when an analytical configuration is formed through a certain order of the notation of entries. For example, it happens within learning the rules of the approximation of numbers. One of the first requirements is familiar to the learners – the given number and the number which is being rounded should be placed under each other, that is – in a column. Then the following plan of action can be used:

– The number written, we will sign zeros under those numbers that will be discarded when rounding. We get a notation (5). We should decide which approximation we are to get – with defect or excess. In a matrix notation we see the digit which is determinative – it is placed above the far left null.

– When performing approximation with excess, we underline the last remaining digit as it is to be changed.

– We apply the rules of approximation and the notation (5) takes the form of (6).

– We put down the rest of the digits of the number and insert the sign of approximate equality. We get the notation (7).

– If within the rounding with excess the last remaining digit is 9, it is expedient to perform notations (8) and (9) instead of (6) and (7) respectively.

$$\begin{array}{rr} 873\,641 & 873\,241 \\ 000 & 000 \end{array} \qquad (5)$$

$$\begin{array}{rr} 87\underline{3}\,641 & 873\,241 \\ 4\,000 & 3\,000 \end{array} \qquad (6)$$

$$\begin{array}{rr} 87\underline{3}\,641\approx & 873\,241\approx \\ \approx 874\,000 & \approx 873\,000 \end{array} \qquad (7)$$

$$\begin{array}{r} 1\underline{29}\,864 \\ 30\,000 \end{array} \qquad (8)$$

$$\begin{array}{r} 1\underline{29}\,864 \approx \\ \approx 130\,000 \end{array} \qquad (9)$$

It is understood that it is only the last notation which will stay in the pupils'/students' notebooks – (7) or (9), as the learners will pass through all phases using the same notation, whilst the teacher within explaining must write down each phase (step) on the blackboard. It should be noted that more complex and more difficult tasks (e.g. – number 43999 is to be rounded to tens) are to be performed later when the learners are fluent in rounding numbers.

Matrix notation of entries also comes in handy when the comparison is made and when new instructional material is being summarized.

For example, within studying the Commutative law of addition and the Commutative law of multiplication [12], special topography of entries enables the learner to independently see common and different features in concrete examples and their generalization.

Commutative Law

Addition	**Multiplication**
$2 + 3 = 3 + 2$,	$2 \cdot 3 = 3 \cdot 2$,
$10 + 15 = 15 + 10$,	$10 \cdot 15 = 15 \cdot 10$,
$0 + 125 = 125 + 0$.	$1 \cdot 125 = 125 \cdot 1$.

In general, for any values of *a* and *b*

$$a + b = b + a \qquad\qquad a \cdot b = b \cdot a$$

It appears possible to more effectively explain to the learners that the Commutative law of addition and the Commutative law of multiplication are similar, almost identical. Therefore, they can be formulated as almost identical sentences. For convenience, the formulation of the Commutative law of addition and the Commutative law of multiplication are combined into one sentence, and the differences are put in brackets:

At rearrangement
of its addends (factors)
a sum (product)
is not changed.

The pupils/students are quick at understanding the specifics of the notation and its deployment in separate texts:

- when they are to read the formulation of the Commutative law of addition, the do not read the words in brackets: *At rearrangement of its addends a sum is not changed*;

- when they are to read the formulation of the Commutative law of multiplication, they do not read the words " addends" which stands for components and "sum" as a result of the operation of addition are omitted and the words in brackets are read: *At rearrangement of its factors a product is not changed*.

6. Conclusions

Teaching students to use content and graphic interpretation of mathematical concepts and facts should be considered as a necessary stage of mathematical training. In doing this students develop visual thinking and enrich their visual and -operational experience which serves as a reliable basis for the development of the verbal-logical thinking of students and creates more ample opportunities for them to express themselves.

Visual and operational experience is required for the students to operate not only the content of geometric instructional material. Visual-operational experience of students is also essential in applying algebraic knowledge, namely – when recoding of, for example, mathematical sentences (equations, inequalities and their systems) in new mathematical sentences is performed, as there is a need in using analytical configurations and updating visual topographic images of the objects of assimilation.

In sum, a balanced selection of the non-verbal shells of the content and their competent use in teaching mathematics should be an object of special care of both – the scholars in the field of the didactics of mathematics, and teachers.

Acknowledgement

The work was supported by the Ministry of Education and Science of Ukraine (state registration number 0115U000639).

References

[1] Veryayev, A.A., *Semiotic approach to education in the informational society*: *Abstract*, Barnaul State Pedagogical University, Barnaul, 2000, 38.

[2] Ilyenkov, E.V, *Dialectical logic: Essays on the History and Theory*, Politizdat, Moscow, 1984, 320.

[3] Lerner, I.Ya., *Didactics of the methods of teaching*, Pedagogics, Moscow, 1981, 186.

[4] Losev, A.F., *The Problem of the Symbol and the Art of Realism*, Art, Moscow, 1976, 367.

[5] *Mathematics in notions, definitions, and terms*: in 2 vol., Volume 2: M – Z, Radyanska Shkola, Kyiv, 1986, 360.

[6] Salmina, N. C, *The sign and symbol in teaching*, Moscow State University Press, Moscow, 1988, 286.

[7] Tarasenkova, N.A., *The theoretic-methodical principles of using of the sign and symbolic means in teaching mathematics of the basic school students: thesis*, Bohdan Khmelnytsky National University of Cherkasy, Cherkasy, 2003, 630.

[8] Tarasenkova, N., "The quality of mathematical education in the context of Semiotics", *American Journal of Educational Research*, 1, no. 11 (2013), 464-471, 2013, *Special issue «Ensuring the quality of higher education»*. Available: http://pubs.sciepub.com/education/1/11/2/index.html.

[9] Tarasenkova, N., "Peculiar Features of Verbal Formulations in School Mathematics," *Global Journal of Human-Social science : G : Linguistics & Education*, Vol. 14, Issue 3, Version 1.0, 61-67, 2014.

[10] Tarasenkova, N.A., "Peculiar features of coding of the geometry notions," *Science and education: a new dimension*, Vol. 5, 7-11, 2013.

[11] Tarasenkova, N.A., "Learning mathematics and semiotics: the point of contact," *Bulletin of Cherkassy University*. *Series: Pedagogical science*, Issue 73, 7-14, 2005.

[12] Tarasenkova, N.A, Bogatyreva, I.M., Bochko, O.P., Kolomiets, O.M., Serdiuk, Z.O., *Mathematics*: *textbook for the 5th form for the secondary schools*, Publishing House "Osvita", Kyiv, 2013, 352.

[13] Tarasenkova, N.A., Bogatyreva, I.M., Kolomiets, O.M., Serdiuk, Z.O., *Mathematics*: *textbook for the 6th form for the secondary schools*, Publishing House "Osvita", Kyiv, 2014, 304.

[14] Tarasenkova, N.A, Bogatyreva, I.M., Bochko, O.P., Kolomiets, O.M., Serdiuk, Z.O., "Conceptual principles of development of text-books on mathematics for 5–6 classes," *Science and education a new dimension*, Vol. 2 (March, 2013), 34-38, 2013.